

Q. $A = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$ char eqⁿ.

$$\det(A - \lambda I) = 0 \quad \lambda^3 - 6\lambda^2 + 6\lambda - 11 = 0$$

Cayley Hamilton theorem: $A^3 - 6A^2 + 6A - 11I = 0$

$$A^3 = 6A^2 - 6A + 11I \quad \downarrow$$

$$A^2 = \begin{bmatrix} 23 & 17 & 1 \\ 0 & 3 & -2 \\ 9 & 7 & -2 \end{bmatrix}$$

$$A^2 - 6A + 6I = 11A^{-1}$$

$$A^{-1} = \frac{1}{11} \begin{bmatrix} 5 & -1 & -7 \\ -4 & 3 & 10 \\ 3 & -5 & -2 \end{bmatrix}$$

Q. from last class:

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} \quad \lambda_1 = -3, \lambda_2 = -3, \lambda_3 = 5$$

$$P = \begin{bmatrix} -2 & 3 & -1 \\ 1 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix}$$

$$P^{-1} = \frac{1}{8} \begin{bmatrix} -2 & 4 & 6 \\ 1 & 2 & 5 \\ -1 & -2 & 3 \end{bmatrix}$$

$$P^{-1}AP = \frac{1}{8} \begin{bmatrix} -2 & 4 & 6 \\ 1 & 2 & 5 \\ -1 & -2 & 3 \end{bmatrix} \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} \begin{bmatrix} -2 & 3 & -1 \\ 1 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= \frac{1}{8} \begin{bmatrix} -2 & 4 & 6 \\ 1 & 2 & 5 \\ -1 & -2 & 3 \end{bmatrix} \begin{bmatrix} 6 & -9 & -5 \\ -3 & 0 & -10 \\ 0 & -3 & 5 \end{bmatrix}$$

$$= \frac{1}{8} \begin{bmatrix} -24 & 0 & 0 \\ 0 & -24 & 0 \\ 0 & 0 & 40 \end{bmatrix} = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

Def: A square matrix A is diagonalizable if there

Def: A square matrix A is diagonalizable if there exists an invertible P s.t. $P^{-1}AP = D$

How to check whether a given matrix is diag.

1) If $\lambda_1, \lambda_2, \dots, \lambda_n$ are all distinct then A is diag

2) If Algebraic multiplicity^{AM} of each eigen value is
= Geometric multiplicity^{GM}, then A is diag-

AM = NO. of times an eigen value is repeating

$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

GM of λ_i = Nullity of $(A - \lambda_i I)$

GM of λ_1 $\geq$ " " $(A + \beta I)$

$$(A + 3I) = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & -6 \\ -1 & -2 & 3 \end{bmatrix}$$

$$^2 \begin{bmatrix} 1 & 2 & -3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \leftarrow \}$$

Nullity of $(A + 3I) = 2 \Rightarrow \text{GM of } \lambda_1 = \lambda_2 = -3$

GM of $\lambda_2 = 5 \Rightarrow$ Nullity of $(A - 5I)$

$$A - 5I = \begin{bmatrix} -7 & 2 & -3 \\ 2 & -4 & -6 \\ -1 & -2 & -5 \end{bmatrix}$$

$$> \begin{bmatrix} 1 & 2 & 5 \\ 2 & -4 & -6 \\ -7 & 2 & -3 \end{bmatrix}$$

$$> \begin{bmatrix} 1 & 2 & 5 \\ 0 & 8 & 16 \\ 0 & 16 & 32 \end{bmatrix}$$

$$> \begin{bmatrix} 1 & 2 & 5 \\ 0 & 8 & 16 \\ 0 & 0 & 0 \end{bmatrix} \leftarrow$$

Nullity of $A - 5I = 1$ GM of $\lambda_3 = 5$ is 1

Eg: $A = \begin{bmatrix} 1 & 2010 & 2011 \\ & -1 & 2010 \\ 0 & 0 & 1 \end{bmatrix}$

$$\lambda_1 = \lambda_2 = 1, \lambda_3 = -1$$

AM of $\lambda_1 = \lambda_2 = 1$ is 2

AM of $\lambda_3 = -1$ is 1

GM of $\lambda_1 = \lambda_2 = 1 \Rightarrow$ Nullity of $(A - I)$

$$> \begin{bmatrix} 0 & 2010 & 2011 \\ 0 & -2 & 2010 \\ 0 & 0 & 0 \end{bmatrix}$$

$$> \begin{bmatrix} 0 & 2010 & 2011 \\ 0 & 0 & 2011 \\ 0 & 0 & 0 \end{bmatrix} \leftarrow$$

GM of $\lambda_1 = \lambda_2 = 1$ is 1.

Note:

$$GM \leq AM$$

H.W:

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \end{bmatrix}$$

H.W.

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix}$$

- 1) char. eqⁿ.
- 2) Eigen values
- 3) Eigen vectors
- 4) P
- 5) $P^{-1}AP = D$
- 6) Cayley Hamilton thm
- 7) A^{-1}
- 8) $AM = GM \rightarrow$ diagonalizable.